Nuclear astrophysics from direct reactions

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Accurate nuclear reaction rates are needed for primordial nucleosynthesis and hydrostatic burning in stars. The relevant reactions are extremely difficult to measure directly in the laboratory at the small astrophysical energies. In recent years direct reactions have been developed and applied to extract low-energy astrophysical S-factors. These methods require a combination of new experimental techniques and theoretical efforts, which are the subject of this presentation.

Keywords: Direct reactions; reactions induced by unstable nuclei; nuclear astrophysics.

1. Challenges in nuclear astrophysics

Ongoing studies in nuclear astrophysics are focused on the opposite ends of the energy scale of nuclear reactions: (a) very high and (b) very low relative energies between nuclei. Projectile with high bombarding energies produce nuclear matter at high densities and temperatures. One expects that matter produced in central nuclear collisions will undergo a phase transition and produce a quark-gluon plasma. One can thus reproduce conditions existing in the first seconds of the universe and also in the core of neutron stars.

At the other end of are the low energy reactions of importance for stellar evolution. Chains of nuclear reactions lead to complicated phenomena like nucleosynthesis, supernovae explosions, and energy production in stars.

1.1. Nuclear reaction rates

Low energy nuclear astrophysics requires the knowledge of the reaction rate $R_{ij}$ between the nuclei $i$ and $j$. It is given by $R_{ij} = n_i n_j \langle \sigma v \rangle / (1 + \delta_{ij})$, where $\sigma$ is the cross section, $v$ is the relative velocity between the reaction partners, $n_i$ is the number density of the nuclide $i$, and $\langle \rangle$ stands for energy average. Extrapolation procedures are often needed to obtain cross sections in the energy region of astrophysical relevance.

While non- resonant cross sections can be rather well extrapolated to the low-energy region, the presence of continuum, or subthreshold resonances, complicates these extrapolations. I will mention few famous examples.

In our Sun the reaction $^7\text{Be}(p, \gamma)^8\text{B}$ plays a major role for the production of high energy neutrinos from the $\beta$-decay of $^8\text{B}$. These neutrinos come directly from the center of the Sun and are ideal probes of the sun’s structure. John Bahcall frequently said that this was the most important reaction in nuclear astrophysics [1]. Our knowledge about this reaction has improved considerably due to new radioactive beam facilities. The reaction $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ is extremely relevant for the fate of massive stars. It determines if the remnant of a supernova explosion becomes a black-hole or a neutron star [2]. These two reactions are only two examples of a large number of reactions which are not yet known with the accuracy needed in astrophysics.

Approximately half of all stable nuclei observed in nature in the heavy element region, $A > 60$, are produced in the $r$-process. This $r$-process occurs in environments with large neutron densities which leads to neutron capture times much smaller than the beta-decay half–lives, $\tau_n \ll \tau_\beta$. The most neutron–rich isotopes along the $r$–process path have lifetimes of less than one second; typically $10^{-2}$ to $10^{-1}$ s. Cross sections for most of the nuclei involved are hard to measure experimentally. Sometimes, theoretical calculations of the capture cross sections as well as the beta–decay half–lives are the only source of input for $r$–process calculations.

1.2. Screening by electrons

Nucleosynthesis in stars is complicated by the presence of electrons. They screen the nuclear charges, therefore increasing the fusion probability by reducing the Coulomb repulsion. Evidently, the fusion cross sections measured in the laboratory have to be corrected by the electron screening when used in a stellar model. This is a purely theoretical problem as one can not reproduce the interior of stars in the laboratory.

A simpler screening mechanism occurs in laboratory experiments due to the bound atomic electrons in the nuclear targets. This case has been studied in great details experimentally, as one can control different charge states of the projectile+target system in the laboratory [3-5]. The experimen-
tual findings disagree systematically by a factor of two with theory. This is surprising as the theory for atomic screening in the laboratory relies on our basic knowledge of atomic physics. At very low energies one can use the simple adiabatic model in which the atomic electrons rapidly adjust their orbits to the relative motion between the nuclei prior to the fusion process. Energy conservation requires that the larger electronic binding (due to a larger charge of the combined system) leads to an increase of the relative motion between the nuclei, thus increasing the fusion cross section. As a matter of fact, this enhancement has been observed experimentally. The measured values are however not compatible with the adiabatic estimate [3-5]. Dynamical calculations have been performed, but they obviously cannot explain the discrepancy as they include atomic excitations and ionizations which reduce the energy available for fusion. Other small effects, like vacuum polarization, atomic and nuclear polarizabilities, relativistic effects, etc., have also been considered [6]. But the discrepancy between experiment and theory remains [5,6].

A possible solution of the laboratory screening problem was proposed by Langanke, Bang, and collaborators [7,8]. Experimentalists often use the extrapolation of the Andersen-Ziegler tables [9] to obtain the average value of the projectile energy due to stopping in the target material. The stopping is due to ionization, electron-exchange, and other atomic mechanisms. However, the extrapolation is challenged by theoretical calculations which predict a lower stopping. Smaller stopping was indeed verified experimentally [5]. At very low energies, it is thought that the stopping mechanism is mainly due to electron exchange between projectile and target. This has been studied in Ref. 10, in the simplest situation: proton+hydrogen collisions. The calculated stopping power was added to the nuclear stopping power mechanism, i.e. to the energy loss by the Coulomb repulsion between the nuclei. The obtained stopping power is proportional to \( v^\alpha \), where \( v \) is the projectile velocity and \( \alpha = 1.35 \). The extrapolations from the Andersen-Ziegler table predict a smaller value of \( \alpha \). Although this result seems to indicate the stopping mechanism as a possible reason for the laboratory screening problem, the theoretical calculations tend to disagree on the power of \( v \) at low energy collisions [11].

Another calculation of the stopping power in atomic \( \text{He}^+ + \text{He} \) collisions using the two-center molecular orbital basis was done in Ref. 12. The agreement with the data from Ref. 11 at low energies is excellent. The agreement with the data disappears if nuclear recoil is included. In fact, the unexpected “disappearance” of the nuclear recoil was also observed in Ref. 13. This seems to violate a basic principle of nature, as the nuclear recoil is due to Coulomb repulsion between projectile and target atoms [9].

2. Direct reactions in/or for nuclear astrophysics

In the previous Sec. I have described a few examples of typical problems in nuclear astrophysics. Now I discuss how direct reactions have been used to attempt solving part of these problems.

2.1. Elastic scattering and \((p, p')\) reactions

The use of internal proton gas targets is a standard technique in radioactive beam facilities. Protons are a very useful probe since their internal structure remains unaffected during low energy collisions. Nuclear densities are a basic input in theoretical calculations of astrophysical reactions at low energies. These can be obtained in, e.g., elastic proton scattering. Elastic scattering in high energy collisions essentially measures the Fourier transform of the matter distribution. Considering for simplicity the one-dimensional case, for light nuclei one has

\[
\int e^{iqx} \rho(x) dx \sim \int e^{iqx}[a^2 + x^2]^{-1} = (\pi/a).e^{-|q|a},
\]

where

\[
q = 2k \sin \theta/2,
\]

for a c.m. momentum \( k \), and a scattering angle \( \theta \). For heavy nuclei the density \( \rho \) is better described by a Fermi function, and

\[
\int e^{iqx}[1 + e^{(x-R)/a}]^{-1} \sim (4\pi).\sin qR.e^{-qa},
\]

for \( R \gg a \), and \( qa \gg 1 \). Thus, the distance between minima in elastic scattering cross sections measures the nuclear size, while its exponential decay dependence reflects the surface diffuseness.

During the last years, elastic proton scattering has been one of the major sources of information on the matter distribution of unstable nuclei in radioactive beam facilities. The extended matter distribution of light-halo nuclei (\(^8\text{He}, ^{11}\text{Li}, ^{11}\text{Be}, \) etc.) was clearly identified in elastic scattering experiments [14,15]. Information on the matter distribution of many nuclei important for the nucleosynthesis in inhomogeneous Big Bang and in r-processes scenarios could also be obtained in elastic scattering experiments. Due to the loosely-bound character and small excitation energies of many of these nuclei, high energy resolution is often necessary.

In \((p, p')\) scattering one obtains information on the excited states of the nuclei. For the same reason as in the elastic scattering case, good accuracy can also be achieved in \((p, p')\) reactions [16].

2.2. Transfer reactions

Transfer reactions \( A(a, b)B \) are effective when a momentum matching exists between the transferred particle and the internal particles in the nucleus. Thus, beam energies should be in the range of a few 10 MeV per nucleon [17]. Low energy reactions of astrophysical interest can be extracted directly from breakup reactions \( A + a \rightarrow b + c + B \) by means of
the Trojan Horse technique as proposed by Baur [18]. If the Fermi momentum of the particle \( x \) inside \( a = (b + x) \) compensates for the initial projectile velocity \( v_0 \), the low energy reaction \( A + x = B + c \) is induced at very low (even vanishing) relative energy between \( A \) and \( x \). To show this, one writes the DWBA cross section for the breakup reaction as

\[
\frac{d^3\Omega_b}{d\Omega_c dE_c} \propto \left| \sum_{lm} T_{lm}(k_a, k_b, k_c) S_{lx} Y_{lm}(k_c) \right|^2,
\]

where

\[
T_{lm} = \langle \chi^{(-)}_b | Y_{lm} f_l | V_{bx} | \chi^{(+)}_a \phi_{bx} \rangle.
\]

The threshold behavior \( E_x \) for the breakup cross section

\[
\sigma_{A+x-B+c} = (\pi/k_x^2) \sum_l (2l + 1) |S_{lx}|^2
\]

is well known: since

\[
|S_{lx}| \sim \exp(-2\pi \eta),
\]

then

\[
\sigma_{A+x-B+c} \sim (1/k_x^2) \exp(-2\pi \eta).
\]

In addition to the threshold behavior of \( S_{lx} \), the breakup cross section is also governed by the threshold behavior of \( f_l(r) \), which for \( r \rightarrow \infty \) is given by

\[
f_l \sim (k_x r)^{1/2} \exp(\pi \eta) K_{2l+1}(\xi),
\]

where \( K_l \) denotes the Bessel function of the second kind of imaginary argument. The quantity \( \xi \) is independent of \( k_x \) and is given by

\[
\xi = (8r/a_B)^{1/2},
\]

where

\[
a_B = h^2/m_Z A Z_x e^2;
\]

is the Bohr length. From this one obtains that

\[
(d^3/\Sigma_b d\Omega_c dE_c)(E_x \rightarrow 0) \approx \text{const}.
\]

The coincidence cross section tends to a constant which will in general be different from zero. This is in striking contrast to the threshold behavior of the two particle reaction \( A+x=B+c \). The strong barrier penetration effect on the charged particle reaction cross section is canceled completely by the behavior of the factor \( T_{lm} \) for \( \eta \rightarrow \infty \). Basically, this technique extends the method of transfer reactions to continuum states. Very successful results using this technique have been reported by Spitaleri and collaborators [19].

Another transfer method, coined as Asymptotic Normalization Coefficient (ANC) technique relies on fact that the amplitude for the radiative capture cross section \( b + x \rightarrow a + \gamma \) is given by

\[
M = \langle I_{bx}^a(r_{bx}) | O(r_{bx}) | \psi_i^{(+)}(r_{bx}) \rangle,
\]

where

\[
I_{bx}^a = \langle \phi_a(\xi_b, \xi_x, r_{bx}) | \phi_x(\xi_x) \phi_b(\xi_b) \rangle,
\]

is the integration over the internal coordinates \( \xi_b \) and \( \xi_x \), of \( b \) and \( x \), respectively. For low energies, the overlap integral \( I_{bx}^a \) is dominated by contributions from large \( r_{bx} \). Thus, what matters for the calculation of the matrix element \( M \) is the asymptotic value of

\[
I_{bx}^a \sim C_{bx}^a W_{-\eta_a, 1/2}(2\kappa_{bx} r_{bx})/r_{bx},
\]

where \( C_{bx}^a \) is the ANC and \( W \) is the Whittaker function. This coefficient is the product of the spectroscopic factor and a normalization constant which depends on the details of the wave function in the interior part of the potential. Thus, \( C_{bx}^a \) is the only unknown factor needed to calculate the direct capture cross section. These normalization coefficients can be found from:

1) Analysis of classical nuclear reactions such as elastic scattering [by extrapolation of the experimental scattering phase shifts to the bound state pole in the energy plane], or

2) peripheral transfer reactions whose amplitudes contain the same overlap function as the amplitude of the corresponding astrophysical radiative capture cross section. This method was proposed by Mukhamezhanov and Timofeyuk [20] and has been used with success for many reactions of astrophysical interest by Tribble and collaborators [21].

To illustrate this technique, let us consider the proton transfer reaction \( A(a, b)B \), where \( a = b + p \), \( B = A + p \). Using the asymptotic form of the overlap integral the DWBA cross section is given by

\[
d\sigma/d\Omega = \sum_{J_B a} \left[ (C_{Ap}^a)^2/\beta_{Ap}^2 \right] [ (C_{bp}^a)^2/\beta_{bp}^2 ] \hat{\sigma}
\]

where \( \hat{\sigma} \) is the reduced cross section not depending on the nuclear structure, \( \beta_{bp} \) (\( \beta_{Ap} \)) are the asymptotic normalization of the shell model bound state proton wave functions in nucleus \( a (B) \) which are related to the corresponding ANC’s of the overlap function as

\[
(C_{bp}^a)^2 = S_{bp}^a \beta_{bp}^2.
\]

Here \( S_{bp}^a \) is the spectroscopic factor. Suppose the reaction \( A(a, b)B \) is peripheral. Then each of the bound state wave functions entering \( \hat{\sigma} \) can be approximated by its asymptotic form and \( \hat{\sigma} \propto \beta_{Ap}^2 \beta_{bp}^2 \). Hence

\[
d\sigma/d\Omega = \sum_{J} (C_{Ap}^a)^2 (C_{bp}^a)^2 R_{Ba}.
\]
where
\[ R_{BA} = \tilde{\sigma} / \beta^2_{Ap} \beta^2_{bp} \]
is independent of \( \beta^2_{Ap} \) and \( \beta^2_{bp} \). Thus for surface reactions the DWBA cross section is actually parameterized in terms of the product of the square of the ANC’s of the initial and the final nuclei \( (C_{i}^{a})^2(C_{f}^{a})^2 \) rather than spectroscopic factors. This effectively removes the sensitivity in the extracted parameters to the internal structure of the nucleus. One of the many advantages of using transfer reaction techniques over direct measurements is to avoid the treatment of the screening problem [19].

### 2.3. Intermediate energy Coulomb excitation

In low-energy collisions the theory of Coulomb excitation is very well understood [22]. But a large number of small corrections are necessary in order to analyze experiments on multiple excitation and reorientation effects. At the other end, the Coulomb excitation of relativistic heavy ions is characterized by straight-line trajectories with impact parameter larger than the sum of the radii of the two colliding nuclei, as shown by Winther and Alder [23].

In first order perturbation theory, the Coulomb excitation cross section is given by
\[ \frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{el} \frac{16\pi^2 Z^2 e^2}{\hbar^2} \times \sum_{\pi\lambda} \frac{B(\pi\lambda, I_i \rightarrow I_f)}{(2\lambda + 1)^3} |S(\pi\lambda, \mu)|^2, \]
where \( B(\pi\lambda, I_i \rightarrow I_f) \) is the reduced transition probability of the projectile nucleus, \( \pi\lambda = E1, E2, M1, \ldots \) is the multipolarity of the excitation, and \( \mu = -\lambda, -\lambda + 1, \ldots, \lambda \).

The orbital integrals \( S(\pi\lambda, \mu) \) contain the information on the dynamics of the reaction [24]. Inclusion of absorption effects in \( S(\pi\lambda, \mu) \) due to the imaginary part of an optical nucleus-nucleus potential where worked out in Ref. 25. These orbital integrals depend on the Lorentz factor \( \gamma = (1 - v^2/c^2)^{-1/2} \), with \( c \) being the speed of light, on the multipolarity \( \pi\lambda\mu \), and on the adiabaticity parameter
\[ \xi(b) = \omega f_i b / \gamma v < 1, \]
where
\[ \omega f_i = (E_f - E_i) / \hbar \]
is the excitation energy (in units of \( \hbar \)) and \( b \) is the impact parameter. Coulomb excitation in radioactive beam facilities are typically performed at bombarding energies of 50-100 MeV/nucleon. It has been very successful to extract precious information of electromagnetic properties of nuclear transitions of astrophysical interest [26]. But a reliable extraction of useful nuclear properties from Coulomb excitation experiments at intermediate energies requires a proper treatment of special relativity [27]. The effect is highly non-linear, i.e. a 10% increase in the velocity might lead to a 50% increase (or decrease) of certain physical observables. A general review of the importance of the relativistic dynamical effects in intermediate energy collisions has been the subject of debate in the literature [27-29].

### 2.4. The Coulomb dissociation method

The (differential, or angle integrated) Coulomb breakup cross section for \( a + A \rightarrow b + c + A \) follows from Eq. 1. It can be rewritten as
\[ \frac{d\sigma^{\pi\lambda}(\omega)}{d\Omega} = F^{\pi\lambda}(\omega; \theta, \phi) \cdot \sigma^{\pi\lambda}_{\gamma a \rightarrow b+c}(\omega), \]
where \( \omega \) is the energy transferred from the relative motion to the breakup, and \( \sigma^{\pi\lambda}_{\gamma a \rightarrow b+c}(\omega) \) is the photo nuclear cross section for the multipolarity \( \pi\lambda \) and photon energy \( \omega \). The function \( F^{\pi\lambda} \) depends on \( \omega \), the relative motion energy, nuclear charges and radii, and the scattering angle \( \Omega = (\theta, \phi) \). \( F^{\pi\lambda} \) can be reliably calculated [24] for each multipolarity \( \pi\lambda \). Time reversal allows one to deduce the radiative capture cross section \( b + c \rightarrow a + \gamma \) from \( \sigma^{\pi\lambda}_{\gamma a \rightarrow b+c}(\omega) \). This method was proposed by Baur, Bertulani and Rebel. Ref. 30. It has been tested successfully in a number of reactions of interest for astrophysics. The most celebrated case is the reaction \( ^7\text{Be}(p, \gamma)^8\text{B} \), first studied by Motobayashi and collaborators [31], followed by numerous experiments in the last decade. A discussion of the results obtained with the method is presented in Ref. 32.

Equation 2 is based on first-order perturbation theory. It also assumes that the nuclear contribution to the breakup is small, or that it can be separated under certain experimental conditions. The contribution of the nuclear breakup has been examined by several authors (see, e.g. [25]). \(^8\text{B} \) has a small proton separation energy \( \approx 140 \text{ keV} \). For such loosely-bound systems it had been shown that multiple-step, or higher-order effects, are important [33]. These effects occur by means of continuum-continuum transitions. The role of higher multipoarities (e.g., E2 contributions [34] in the reaction \( ^7\text{Be}(p, \gamma)^8\text{B} \)) and the coupling to high-lying states has also to be investigated carefully.

### 2.5. Charge exchange reactions

During supernovae core collapse, temperatures and densities are high enough to ensure that nuclear statistical equilibrium is achieved. This means that for sufficiently low entropies, the matter composition is dominated by the nuclei with the highest binding energy for a given \( Y_e \). Electron capture reduces \( Y_e \), driving the nuclear composition to more neutron rich and heavier nuclei, including those with \( N > 40 \), which dominate the matter composition for densities larger than a few \( 10^{10} \text{ g cm}^{-3} \). As a consequence of the model applied...
in collapse simulations, electron capture on nuclei ceases at these densities and the capture is entirely due to free protons. To understand the whole process it is necessary to obtain Gamow-Teller matrix elements which are not accessible in beta-decay experiments. Many-body theoretical calculations are right now the only way to obtain the required matrix elements. This situation can be remedied experimentally by using charge-exchange reactions.

Charge exchange reactions induced in $(p, n)$ reactions are often used to obtain values of Gamow-Teller matrix elements, $B(GT)$, which cannot be extracted from beta-decay experiments. This approach relies on the similarity in spin-isospin space of charge-exchange reactions and $\beta$-decay operators. As a result of this similarity, the cross section $\sigma(p, n)$ at small momentum transfer $q$ is closely proportional to $B(GT)$ for strong transitions [35]. Taddeucci’s formula reads

$$\frac{d\sigma}{dq}(q = 0) = KN_D|J_{\sigma\tau}|^2B(\alpha),$$

(3)

where $K$ is a kinematical factor, $N_D$ is a distortion factor (accounting for initial and final state interactions), $J_{\sigma\tau}$ is the Fourier transform of the effective nucleon-nucleon interaction, and $B(\alpha = F, GT)$ is the reduced transition probability for non-spin-flip,

$$B(F) = (2J_i + 1)^{-1}|\langle f||\sum_k \tau_k^{(\pm)}||i\rangle|^2,$$

and spin-flip,

$$B(GT) = (2J_i + 1)^{-1}|\langle f||\sum_k \sigma_k \tau_k^{(\pm)}||i\rangle|^2,$$

transitions.

Taddeucci’s formula, valid for one-step processes, was proven to work rather well for $(p, n)$ reactions (with a few exceptions). For heavy ion reactions the formula might not work so well. This has been investigated in Refs. 36 and 37. In Ref. 36, it was shown that multistep processes involving the physical exchange of a proton and a neutron can still play an important role up to bombarding energies of 100 MeV/nucleon. Ref. 37, use the isospin terms of the effective interaction to show that deviations from the Taddeucci formula are common under many circumstances. As shown in ref. 38, for important GT transitions whose strength are a small fraction of the sum rule the direct relationship between $\sigma(p, n)$ and $B(GT)$ values also fails to exist. Similar discrepancies have been observed [39] for reactions on some odd-A nuclei including $^{13}$C, $^{15}$N, $^{35}$Cl, and $^{39}$K and for charge-exchange induced by heavy ions [40]. It is still an open question if Taddeucci’s formula is valid in general.

Undoubtedly, charge-exchange reactions such as $(p, n)$, $(^3$He,t) and heavy-ion reactions (A, A±1) can provide information on the $B(F)$ and $B(GT)$ values needed for astrophysical purposes. This is one of the most research areas in radioactive beam facilities and has been used successfully by Austin, Zegers, and collaborators [41].

2.6. Knock-out reactions

Exotic nuclei are the raw materials for the synthesis of the heavier elements in the Universe, and are of considerable importance in nuclear astrophysics. Modern shell-model calculations are now able to include the effects of residual interactions between pairs of nucleons, using forces that reproduce the measured masses, charge radii and low-lying excited states of a large number of nuclei. For very exotic nuclei the small additional stability that comes with the filling of a particular orbital can have profound effects upon their existence as bound systems, their lifetimes and structures. Thus, verifications of the ordering, spacing and the occupancy of orbitals are essential in assessing how exotic nuclei evolve in the presence of large neutron or proton imbalance and our ability to predict these theoretically. Such spectroscopy of the states of individual nucleons in short-lived nuclei uses direct nuclear reactions.

Single-nucleon knockout reactions with heavy ions, at intermediate energies and in inverse kinematics, have become a specific and quantitative tool for studying single-particle occupancies and correlation effects in the nuclear shell model, as described by Hansen and Tostevin [42,43]. The experiments observe reactions in which fast, mass $A$, projectiles collide peripherally with a light nuclear target producing residues with mass $(A - 1)$ [43]. The final state of the target and that of the struck nucleon are not observed, but instead the energy of the final state of the residue can be identified by measuring coincidences with decay gamma-rays emitted in flight.

New experimental approaches based on knockout reactions have been developed and shown to reduce the uncertainties in astrophysical rapid proton capture (rp) process calculations due to nuclear data. This approach utilizes neutron removal from a radioactive ion beam to populate the nuclear states of interest. In the first case studied by Schatz and collaborators [44], $^{33}$Ar, excited states were measured with uncertainties of several keV. The 2 orders of magnitude improvement in the uncertainty of the level energies resulted in a 3 orders of magnitude improvement in the uncertainty of the calculated $^{32}$Cl$(p,\gamma)^{33}$Ar rate that is critical to the modeling of the rp process. This approach has the potential to measure key properties of almost all interesting nuclei on the rp-process path.

3. Reconciling nuclear structure with nuclear reactions

Many reactions of interest for nuclear astrophysics involve nuclei close to the dripline. To describe these reactions, a knowledge of the structure in the continuum is a crucial feature. Recent works [45, 46] are paving the way toward a microscopic understanding of the many-body continuum. A basic theoretical question is to what extent we know the form of
the effective interactions for threshold states. It is also hope-
less that these methods can be accurate in describing high-
lying states in the continuum. In particular, it is not worth-
while to pursue this approach to describe direct nuclear re-
actions.

A less ambitious goal can be achieved in the coming years
by using the Resonating Group Method (RGM) or the Gener-
ator Coordinate Method (GCM). These form a set of coupled
integro-differential equations of the form

$$\sum_{\alpha'} \int d^3 r' \left[ H^{AB}_{\alpha\alpha'}(r, r') - E N^{AB}_{\alpha\alpha'}(r, r') \right] g^{\alpha}_{\alpha'}(r') = 0,$$  \hspace{1cm} (4)

where

$$H^{AB}_{\alpha\alpha'}(r, r') = \langle \Psi_A(\alpha, r)|H|\Psi_B(\alpha', r') \rangle,$$

and

$$N^{AB}_{\alpha\alpha'}(r, r') = \langle \Psi_A(\alpha, r)|\Psi_B(\alpha', r') \rangle.$$  

In these equations $H$ is the Hamiltonian for the system of
two nuclei (A and B) with the energy $E$, $\Psi_{A, B}$ is the wave-
function of nucleus (A and B), and $g^{\alpha}_{\alpha'}(r)$ is a function to
be found by numerical solution of Eq. (4), which describes
the relative motion of A and B in channel $\alpha$. Full anti-
symmetrization between nucleons of A and B are implicit.
Modern nuclear shell-model calculations, including the
No-Core-Shell-Model (NCSM) are able to provide the wave-
functions $\Psi_{A, B}$ for light nuclei. But the Hamiltonian in-
volves an effective interaction in the continuum between the
clusters A and B. It is very hard, if not impossible, to ob-
tain this effective interaction within microscopic models. Old
tools, such as parameterized phenomenological interactions  
(e.g. M3Y [47]) are still the only way to access effective in-
teraction for high energy nucleus-nucleus scattering.

Overlap integrals of the type $I_{A\alpha}(r) = \langle \Psi_{A, \alpha'}|\Psi_A \rangle$ for
bound states has been calculated by Navrattil [48] within
the NCSM. This is one of the inputs necessary to calculate S-factors for radiative capture, $S_{\alpha} \sim |\langle \Psi_{\alpha} | H_{EM} | I_{A \alpha} \rangle|^2$, where $H_{EM}$ is a corresponding electromagnetic operator.  
The left-hand side of this equation is to be obtained by solv-
ing Eq. (4). For some cases, in particular for the $p^{+7}$Be re-
action, the distortion caused by the microscopic structure of
the cluster does not seem to be crucial to obtain the wave-
function in the continuum. The wavefunction is often ob-
tained by means of a potential model. The NCSM overlap
integrals, $I_{A\alpha}$, can also be corrected to reproduce the right
asymptotics [49], given by

$$I_{A\alpha}(r) \propto W_{-\eta, l+1/2}(2k_0 r),$$

where $\eta$ is the Sommerfeld parameter, $l$ the angular momentum,
$k_0 = \sqrt{2\mu E_0}/\hbar$, with $\mu$ the reduced mass and $E_0$ the
separation energy.

A step in the direction of reconciling structure and re-
actions for the practical purpose of obtaining astrophysical
S-factors, along the lines described in the previous paragraph,
was obtained in Ref. 49 and 50. The wavefunctions ob-
tained in this way were shown to reproduce very well the
momentum distributions in knockout reactions of the type
$^{8}\text{B} + A \rightarrow ^7\text{Be} + X$ obtained in experiments at MSU and
GSI facilities. The astrophysical S-factor for the reaction
$^7\text{Be}(p, \gamma)^8\text{B}$ was also calculated and excellent agreement was
found with the experimental data in both direct and indirect
measurements [49,50]. The low- and high-energy slopes of
the S-factor obtained with the NCSM is well described by the fit

$$S_{17}(E) = (22.109 \text{ eV.b}) \times \frac{1 + 5.30 E + 1.65 E^2 + 0.857 E^3}{1 + E/0.1375},$$  \hspace{1cm} (5)

where $E$ is the relative energy (in MeV) of $p^{+7}$Be in their
center-of-mass. This equation corresponds to a Padé approx-
imation of the S-factor. A subthreshold pole due to the binding
energy of $^8\text{B}$ is responsible for astrophysics. The ELISE experiment setup will use
electrons scattered off radioactive nuclei. These experiments
will explore an unknown world of studies with nuclei far from
stability which play an important role in our universe.

It was shown [53] that for the conditions attained in the
electron-ion collider mode, the electron scattering cross sec-
tions are directly proportional to photonuclear processes with
real photons. This proportionality is lost when larger scatter-
ing angles, and larger ratio of the excitation energy to the
electron energy, $E_\gamma/E$, are involved. One of the important
issues to be studied in future electron-ion colliders is the nu-
clear response at low energies. This response can be modeled
in two ways: by a (a) direct breakup and by a (a) collective
excitation. In the case of direct breakup the response function
will depend quite strongly on the final-state interaction [53].
This may become a very useful technique to obtain phase
shifts, or effective-range expansion parameters, of fragments
far from the stability line.

The electromagnetic response of light nuclei, leading to
their dissociation, has a direct connection with the nuclear
physics needed in several astrophysical sites. In fact, it has
been shown [54] that the existence of pygmy resonances have
important implications on theoretical predictions of radiative
neutron capture rates in the r-process nucleosynthesis and
consequently on the calculated elemental abundance distri-
bution in the universe.
The US needs urgently a new radioactive beam facility, fully dedicated to the physics of radioactive nuclei. Without competing facilities worldwide, observational and theoretical astrophysics will never be able to constrain numerous models used to understand our universe.

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